Consecutive Systems Asymptotic Threshold Behaviors

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Instrumente Structurale
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Context

Two-terminal network reliability

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1. Design two-terminal networks highly reliable

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2. Algorithms for computing the reliability polynomial of a two-terminal network

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Two-terminal network reliability

1. Design two-terminal networks highly reliable
2. Algorithms for computing the reliability polynomial of a two-terminal network
3. Study their properties (graph structure, asymptotic behavior, etc.)

Context

What are the possible solutions

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1. von Neuman ‘52 – parallel structures with majority voting/multiplexing

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1. von Neuman ‘52 – parallel structures with majority voting/multiplexing
2. Moore and Shannon ‘56 – define minimal two-terminal circuits and introduce hammock networks/circuits

Reliability

It comes as no surprise that hundreds of seemingly natural definitions arise by examining the plethora of different types of networks, causes and types of failures, and levels and types of operation. One should not expect to find a single definition for reliability that accommodates the many real situations of importance.

(Charles J. Colbourn)

Reliability

- Colbourn, Barlow and Proschan, Ball and Provan – Network theory

    The connectivity of the input/source with the output/terminus

Reliability

- Colbourn, Barlow and Proschan, Ball and Provan – Network theory

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- Moore and Shannon – Information theory

Reliability

- Colbourn, Barlow and Proschan, Ball and Provan – Network theory

  The connectivity of the input/source with the output/terminus

- Moore and Shannon – Information theory

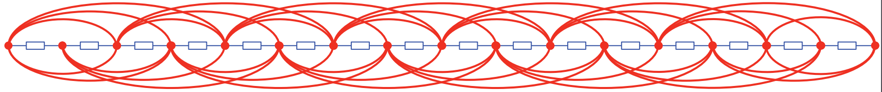
  Connectivity and non-connectivity

Moore and Shannon A large improvement in reliability, both when the coil is energized and when it is not energized

Consecutive systems

Consecutive systems

A consecutive system, better known as a consecutive-k-out-of-n :F system, is defined as a system having n components placed in a row (i.e., sequentially), which fails if and only if at least k consecutive components fail (Kontoleon 1980)



Relaibility polynomial

Exact formula

- de Moivre, Uspensky

$$B_{n,k} = \sum_{j=0}^{\lfloor \frac{n}{k+1} \rfloor} (-1)^j \binom{n-jk}{j} (pq^k)^j$$

$$Rel(k,n;q) = B_{n,k} - q^k B_{n-k,k}$$

Relaibility polynomial
Exact formula

Proposition 1 (Fu 1987)

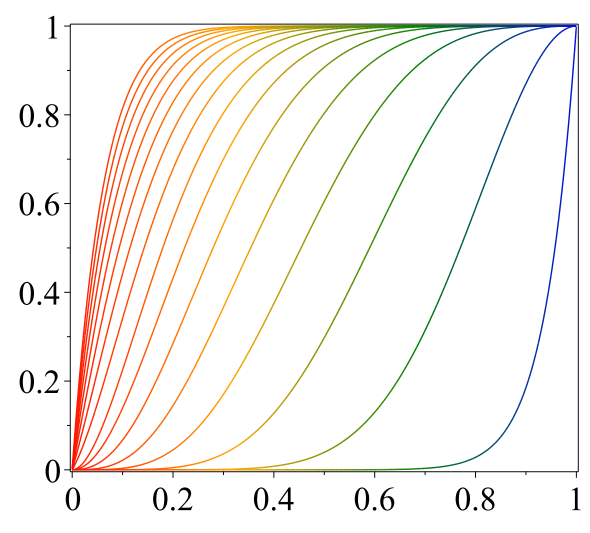
Let M be a square matrix of size $k+1$ given by

$$M = \begin{pmatrix} p & q & 0 & \dots & 0 \\ p & 0 & q & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p & 0 & 0 & \dots & q \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix}.$$

Then

$$Rel(k, n; p) = (1, 0, \dots, 0) \times M^n \times (1, \dots, 1, 0)^t. \qquad (1)$$

Consecutive systems



Consecutive systems

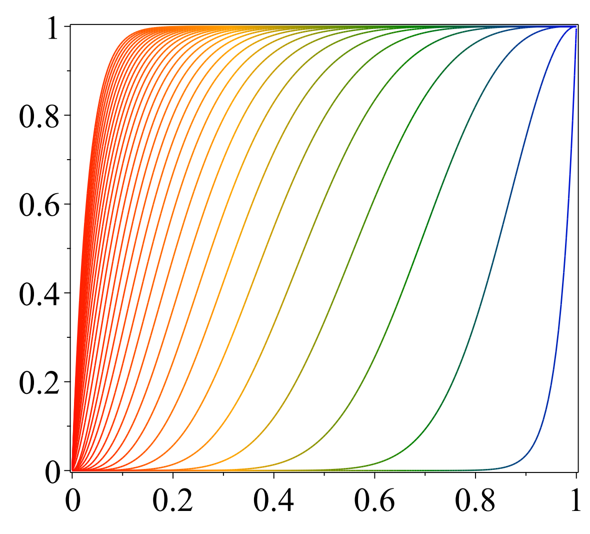


Figure – 32

Consecutive systems

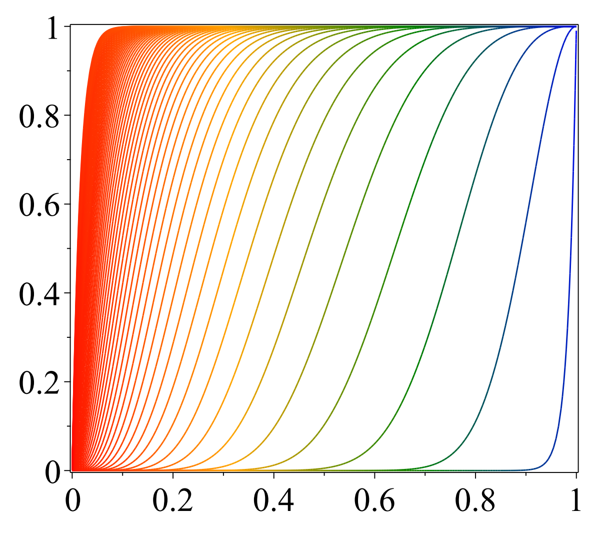


Figure – 64

Consecutive systems

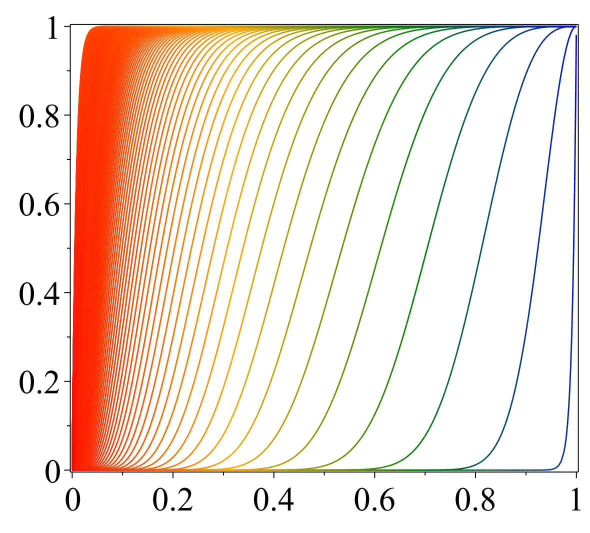


Figure – 128

Consecutive systems

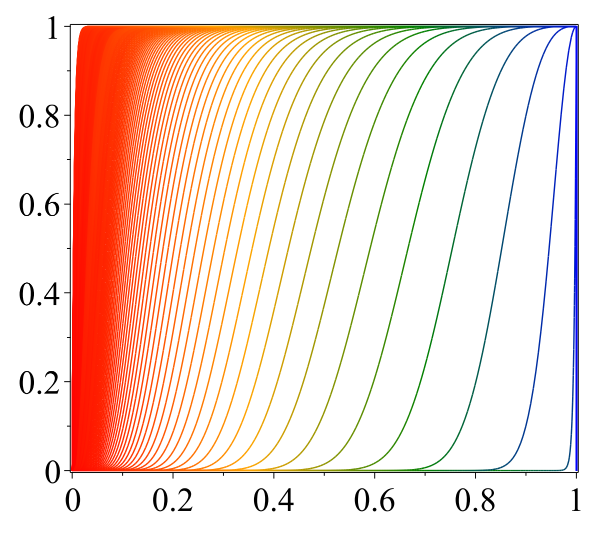


Figure – 256

Consecutive systems

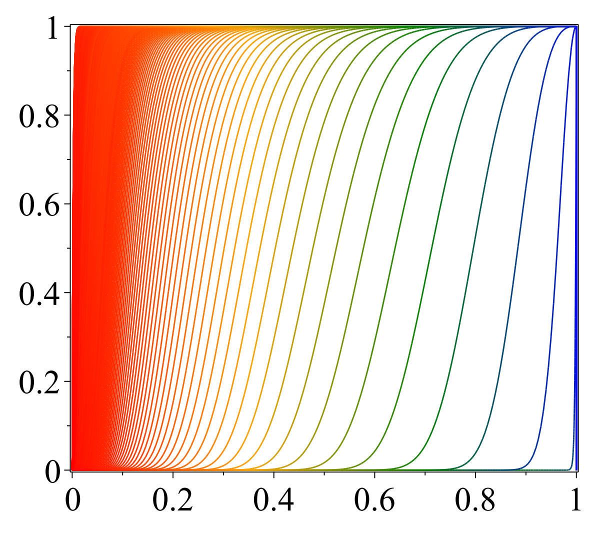


Figure – 512

Consecutive systems

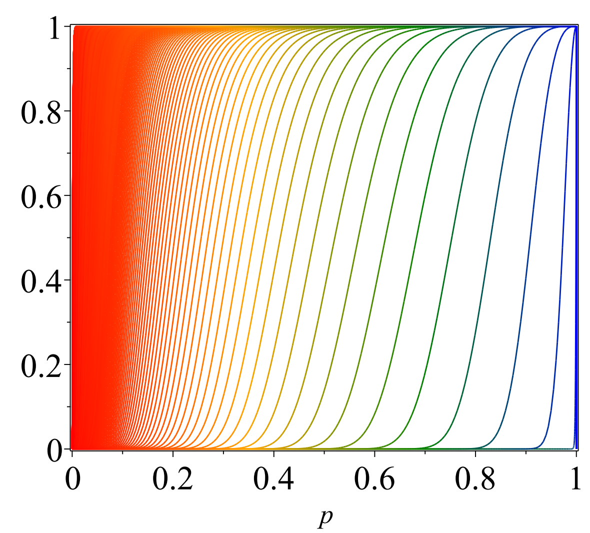
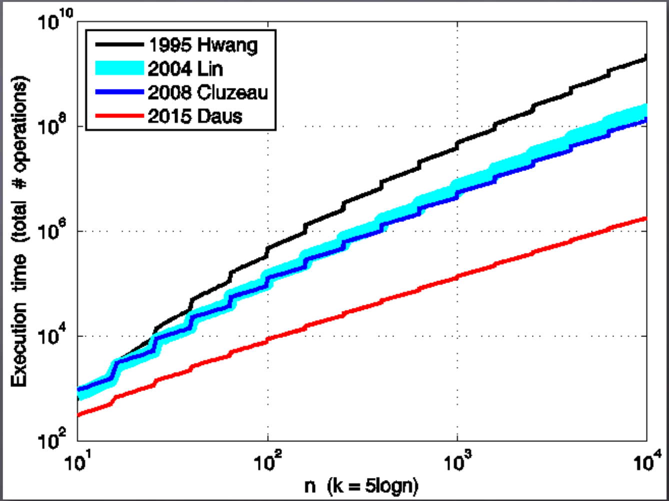


Figure – 1024

Consecutive systems
Complexity



Average Reliability

- How reliable a consecutive network is in average ?

- Complexity challenge : compute $Rel(k, n; p)$ and then take the integral.

- Asymptotics when $n \to \infty$

Average Reliability

Exact formula

Theorem 1

Let n be a strictly positive integer. Then for any $\lfloor \frac{n}{r} \rfloor \leqslant k < \lfloor \frac{n}{r-1} \rfloor$, the average reliability of a consecutive-k-out-of-n :F system equals

$$1 + (n+2) \sum_{j=1}^{r-1} (-1)^j \frac{\binom{n-jk}{j-1}}{(jk+1)(jk+2)\binom{jk+j+1}{j-1}} \tag{2}$$

Average Reliability

Exact formula

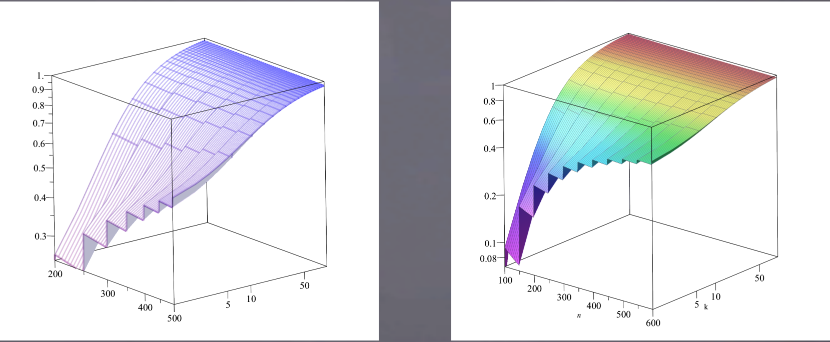
- $k > n/2$

$$Avr_2(n,k) = 1 - \frac{n+2}{(k+1)(k+2)}$$

- $k > n/3$

$$Avr_3(n,k) = Avr_2(n,k) + \frac{(n+2)(n-2k)}{(2k+1)(2k+2)(2k+3)}$$

Consecutive systems



(a) (b)

Figure – Average reliability of consecutive systems

Consecutive systems
Asymptotics formulae

k	n/2	n/3	n/4	n/5	n/6
$Avr(n,k)$	$1-\frac{4}{n+4}$	$1-\frac{7.8}{n}+O(\frac{1}{n^2})$	$1-\frac{12.1}{n}+O(\frac{1}{n^2})$	$1-\frac{16.8}{n}+O(\frac{1}{n^2})$	$1-\frac{21.7}{n}+O(\frac{1}{n^2})$

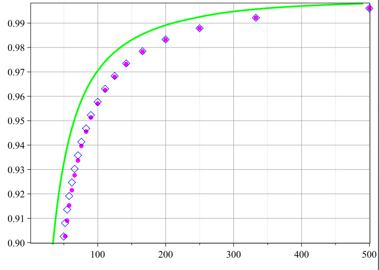
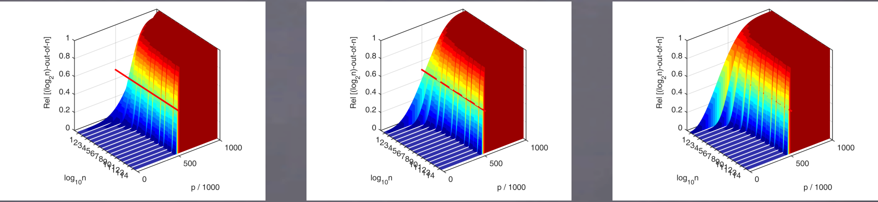


Figure – $Avr(n,k)$, first and second term approximation ($n=1000$).

Consecutive systems

Conjecture



(a) $k = \log_2 n - 1$ (b) $k = \log_2 n$ (c) $k = \log_2 n + 1$

Figure – Average reliability of consecutive systems

Funding

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